The correct interpretation of the tensile strength of short fibre-reinforced composites

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An analytical method of calculating the tensile strength of composites, with perfectly bonded and randomly oriented short fibres, was described recently in this journal by Zhu *et al.* [1]. When comparing the calculations with experimental results for aluminum alloys, reinforced with Al_2O_3 fibres in a three-dimensionally random array, the conclusions were incorrect. The authors did not recognize that in this type of composite, the transverse fibres often delaminate. This paper describes an alternative and simpler method of calculating the tensile strength, which includes both perfect bonding and the effect of delamination. This method has been applied previously to two-dimensional random systems and is here extended to three-dimensional systems. The calculated values of strength are in excellent agreement with the experimental values quoted by Zhu *et al.* Further support is provided by comparisons with more extensive data for other metal matrix composites. General conclusions for a three-dimensional random system are that: (i) perfect bonding will only provide a modest increase of strength; and (ii) delamination of the transverse fibres will drastically reduce the strength and cannot be tolerated.

1. Introduction

One of the primary advantages of composites containing discontinuous, or short, fibres is the increase of tensile strength endowed by the reinforcements. In this regard the orientation of the fibres is particularly important, and to obtain isotropic properties the fibres are randomly oriented either three-dimensionally or in a planar two-dimensional array. The tensile strength of such a system can be calculated from a knowledge of the properties of the components: i.e. fibre, matrix and interface. Recently Zhu *et al.* [1] have proposed a new analytical method, which includes not only the direct mechanical contribution of the fibres, but also other factors such as the residual stresses and strengthening mechanisms in the matrix, which are difficult to quantify. However, from comparisons with some experimental data from the literature [2], they conclude that the direct mechanical contribution of the fibres is the dominant component. This is certainly true for most systems except, as they point out, for the extensive work hardening of a pure aluminium matrix. However, some of their other conclusions are invalid and are the motivation for this paper.

Zhu *et al.* [1] focused their attention on three-dimensional (3D) randomly oriented systems, and from the rather limited experimental data available, they selected two examples [2] of aluminium–silicon alloys reinforced by Al_2O_3 fibres (Saffil [3]). One example is an aluminium–12 Si alloy with 25% volume fraction fibres having a tensile strength of 163 MPa (165 MPa in the original reference). Their theory attributes 83 MPa directly to the fibres and the remaining 80 MPa to the matrix. But they fail to mention that the unreinforced alloy has a tensile strength of 143 MPa! Thus, we are expected to believe that the addition of the fibres has somehow reduced the matrix alloy strength by 63 MPa. Such an effect would be difficult to explain other than by invoking serious macroscopic defects, such as incomplete metal infiltration of the fibrous preform during casting. Alternatively, their calculation of the direct contribution of the fibres is incorrect.

The second example considered by Zhu *et al.* is an aluminium–7 Si alloy reinforced with 20% volume fraction of Saffil fibres having a tensile strength of 237 MPa [2]. Their interpretation of this result focuses on the need for thermal stress induced dislocations in the matrix. But they totally ignore the simple fact that the unreinforced alloy had a tensile strength of 312 MPa [2]; i.e. the addition of the fibres resulted in a substantial reduction of strength. Their model does not explain this result and their interpretation is totally wrong.

Before the paper by Zhu *et al.* is quoted by others, it is important to correct the above fallacies. This can be accomplished very simply. Firstly, it must be recognized that the interfacial bond between alumina fibres and aluminium alloys is not always perfect (as assumed by Zhu *et al.*), so that failures may be initiated by delamination of the transverse fibres, i.e. those aligned perpendicularly to the stress direction [4]. (The importance of this factor, and the need to characterize the fracture mode, will be described in more detail in a paper currently in preparation.) Secondly, several years ago this author

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[5] described a simple model for calculating the tensile strength of randomly oriented short fibre composites, which accounted for this delamination as well as the perfectly bonded system. At that time the model was applied very successfully to two-dimensional systems, and shown to be in good agreement with a variety of experimental data. It is now opportune to apply this model to 3D systems, such as those considered by Zhu *et al*.

2. Calculation of the ultimate tensile strength

The method of calculating the UTS was described in detail previously for a two-dimensional (2D) system [5], so only an outline is given here. It is based upon the anisotropy of the strength σ (θ) of a composite with aligned continuous (long) fibres, which is described by the Tsai–Hill equation [6]

$$\sigma(\theta) = \left[\frac{\cos^4\theta}{\sigma_L^2} + \left[\frac{1}{\tau^2} - \frac{1}{\sigma_L^2}\right]\sin^2\theta\cos^2\theta + \frac{\sin^4\theta}{\sigma_T^2}\right]^{-1/2} \quad (1)$$

where θ is the angle between the fibres and the direction of the applied load, σ_L is the strength when $\theta = 0^\circ$, σ_T is the strength when $\theta = 90^\circ$, and τ is the shear strength of the composite.

This relationship is applied to a short fibre composite by substituting the following well-known equations [7] for σ_L

$$\sigma_L = V_\mathrm{f}\sigma_\mathrm{f}\left[1 - \frac{\ell_c}{2\ell}\right] + [1 - V_f]\sigma_m \qquad \text{for } \ell \geq \ell_c \quad (2)$$

or

$$\sigma_L = V_\mathrm{f}\sigma_\mathrm{f}\frac{\ell}{2\ell_c} + [1 - V_f]\sigma_m \qquad \text{for } \ell \leq \ell_c$$

where V_f is the volume fraction of the fibre, σ_f is the strength of the fibre, ℓ is the length of the fibre, σ_m is the strength of the matrix and ℓ_c, the so-called critical or ineffective length is given by

$$\frac{\ell_c}{d} = \frac{\sigma_f}{2\tau_i} \quad (3)$$

where d is the fibre diameter and τ_i is the shear strength of the fibre matrix interface.

To calculate the upper limit of strength, it is assumed that a strong interfacial bond is formed so that both τ and τ_i will be maximized, namely they will be equal to the shear strength of the matrix (τ_m)

$$\tau = \tau_i = \tau_m \quad (4)$$

The value of τ_m is set equal to $0.67\sigma_m$, which is typical for aluminium alloys [8].

Similarly, the maximum value of the transverse strength (σ_T) will be the tensile strength of the matrix, which is taken to be equal to that of the unreinforced alloy

$$\sigma_\mathrm{T}(\max) = \sigma_\mathrm{m} \quad (5)$$

Thus the maximum strength (σ_c (max)) of a randomly oriented short fibre composite is calculated by substituting Equations 2–5 in to Equation 1 and integrating. For a 2D planar random system

$$\sigma_c = \frac{2}{\pi}\int_0^{\pi/2}\sigma(\theta)\,\mathrm{d}\theta \quad (6)$$

while for a random system

$$\sigma_c = \int_0^{\pi/2}\sigma(\theta)\sin\theta\,\mathrm{d}\theta \quad (7)$$

For randomly oriented fibres, the assumption of a strong interfacial bond is most likely to break down for the fibres oriented perpendicularly to the applied stress. If delamination occurs at a stress $\sigma_i < \sigma_m$, then Equation 5 no longer applies. In fact, when such delamination occurs in an aligned composite, the transverse strength can be approximated by representing each fibre as a cylindrical hole in the matrix [9], i.e.

$$\sigma_T = \sigma_m\left[1 - 2(V_f/\pi)^{1/2}\right] \quad (8)$$

Substitution of this expression in lieu of Equation 5 yields a lower limit of the composite strength (σ_c (min)).

3. Comparison with experiment

3.1. Saffil/Aluminium

It is useful to begin with the general predictions of our model for composites containing Saffil fibres. These fibres have a strength of 2 GPa, an average diameter of 3 μm and are initially 500 μm in length [2, 3]. However, after processing into a composite, the average length is reduced typically to 90 μm [10], so we will use this value. The example in Fig. 1 shows the effect of fibre volume fraction on the tensile strength for a matrix strength of 300 MPa, which is typical of aluminium alloys at 20 °C. As expected, there is a large difference in strength between the 2D and 3D systems. (However, the 3D strengthening is isotropic, whereas the 2D

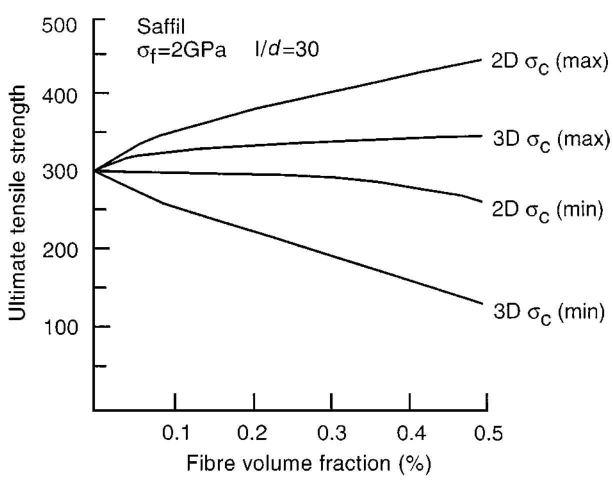

Figure 1 The effect of the volume fraction of Saffil fibres on the calculated values of ultimate tensile strength, for composites with a matrix strength typical of aluminium alloys. The curves correspond to perfect bonding σ_c (max) and transverse fibre delamination σ_c (min) for 2D and 3D randomly oriented fibres.

TABLE I The ultimate tensile strength of two aluminium alloys reinforced with Saffil fibres randomly oriented in three dimensions. Experimental values from Friend [2]. Calculated values for perfect bonding and delamination of the transverse fibres

Alloy	Fibre volume fraction (%)	Tensile strength (MPa)		
			Calculated	
		Experimental	σ_c (max)	σ_c (min)
Al–12 Si	0	143	—	—
Al–12 Si	25	165	167	107
Al–7 Si	0	312	—	—
Al–7 Si	20	237	349	235

system is strengthened only in the plane of the fibres.) For a 2D system the strength is increased substantially by perfectly bonded fibers, but if the transverse fibres delaminate, the strength is virtually independent of the fibre volume fraction. For a 3D system strongly bonded fibres provide a modest increase of strength, but if the transverse fibres delaminate the strength decreases with increasing fibre volume fraction.

The experimental results considered by Zhu *et al.* can now be explained correctly. The Al–12 Si alloy was strengthened by the Saffil fibres, indicating a strong interfacial bond. The Al–7 Si alloy was weakened by the addition of Saffil fibres, as would result from delamination of the transverse fibres. This interpretation is confirmed quantitatively as summarized in Table I: the strength of the Al–12 Si composites agrees with the calculated value of σ_c (max), while that of the Al–7 Si composite corresponds to the calculated value of σ_c (min). Further, this excellent agreement between the calculated and measured values of tensile strength is obtained without recourse to other factors such as residual stresses, dislocation densities, etc. showing that they are not significant. Thus, we agree with Zhu *et al.* that the dominant component of strengthening is the direct load sharing mechanical contribution of the fibres, but it is absolutely essential to characterize the mode of failure by fractography and to incorporate this information into any model calculation of tensile strength.

It is noteworthy that the experimental results quoted above [2] were obtained from composites formed by liquid metal infiltration of a Saffil fibre preform. Thus the arrangement of the fibres is dictated by the method of preform fabrication, and the fibre distribution often approximates a 2D planar array. However Friend [2] examined the Saffil preforms with a scanning electron microscope and characterized the fibre distribution as approximating to a 3D random system. This observation is consistent with the analysis presented here. On the other hand, if for example, the Al–7Si alloy is reinforced with a 2D array of fibres, then examination of Fig. 1 shows that the tensile strength will either increase substantially (2D σ_c(max)) or be essentially unchanged if delamination occurs (2D σ_c(min)); both these options are contrary to the experimental result (Table I).

3.2. Other aluminium composites

To add credence to the above analysis of two individual composites it is appropriate to evaluate other more systematic studies. This has been done previously for 2D random systems [5], where the experimental data is more plentiful than for 3D random systems. However, we are aware of three studies of 3D composites in which the fibre properties σ_f, V_f and ℓ/d are specified, and where the unreinforced alloy has been processed in the same manner as the composite.

Jones and Wawner [11] reinforced 332 aluminium with chopped FP Al_2O_3 fibres ($\sigma_f = 1.4$ GPa and $\ell/d = 10$), producing a 3D random system by compocasting. Some of their results are plotted in Fig. 2, which shows the effect of fibre volume fraction on the tensile strength. At both 20 °C and 370 °C the fibres confer only a modest increase in strength. The two curves show the calculated values of σ_c (max) and are in excellent agreement with the experimental data. Thus we conclude that a strong interfacial bond was achieved (i.e. $\sigma_i > \sigma_m$). Note that the modest degree of strengthening is the most that can be realized with this system, because the critical length ℓ_c is comparable to the fibre length (ℓ).

The results of Das *et al.* [12] for Nicalon ($\sigma_f = 3$ GPa, $\ell/d = 20$) reinforcement of a zinc alloy are plotted in Fig. 3, and are of particular interest because at 200 °C

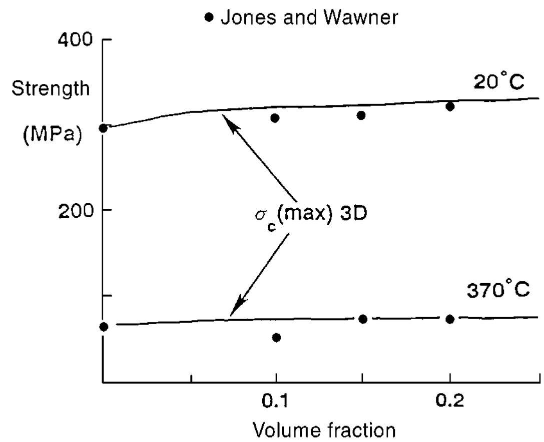

Figure 2 Effect of volume fraction of FP Al_2O_3 on the strength of 332 aluminium. The experimental data of Jones and Wawner [11] (●) are compared with the calculated values of σ_c (max) for a 3D random system with perfect bonding.

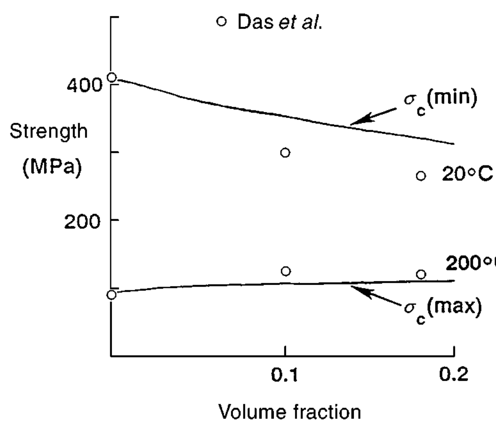

Figure 3 Effect of volume fraction of Nicalon fibres on the strength of a zinc alloy. The experimental data of Das *et al.* [12] are compared with calculated values for a 3D random system.

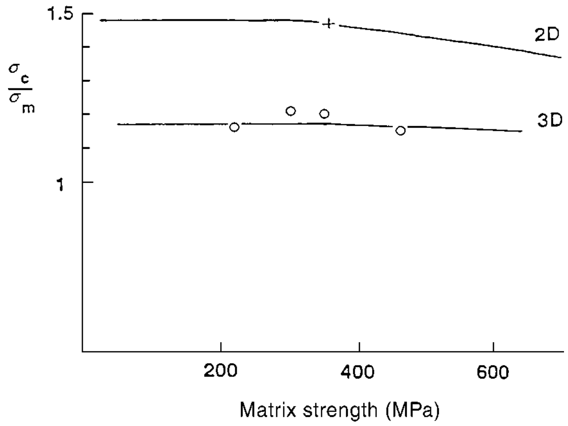

Figure 4 The effect of the strength of the unreinforced matrix alloy (σ_m) on the strength of a composite (σ_c) with 20% SiC whiskers. Experimental data from Morimoto *et al.* [13] (+) and Sachdev and Gerard [14] (○). Lines are calculated values for 2D and 3D random systems with perfect bonding.

a modest increase of strength was achieved, while at 20 °C the strength decreased quite dramatically. These opposing trends can be accounted for by our model. At 200 °C the matrix is very weak so that $\sigma_i > \sigma_m$ and the calculated values of σ_c (max) are in good agreement with the experimental data. But at 20 °C Das *et al.* observed that the evidence for a good interfacial bond was "not clear cut". Therefore, we assume that the perpendicularly oriented fibres debonded, i.e. $\sigma_i < \sigma_m$, and calculate the value of σ_c (min). As shown in Fig. 3, the weakening effect of the fibres is predicted, but the experimental values are even smaller than σ_c (min). This suggests that, in this case, the fibre–matrix bond was so weak that the interfacial shear strength was less than the matrix shear strength, i.e. Equation 4 does not hold.

Finally, the 3D and 2D random systems can be compared for aluminium alloys reinforced with SiC whiskers ($\sigma_f = 8$ GPa, $\ell/d = 20$ and $V_f = 20\%$). This type of composite has been fabricated by powder metallurgy to produce a 2D random system by Morimoto *et al.* [13], while 3D random systems have been produced by squeeze casting by Sachdev and Gerard [14]. This data is summarized in Fig. 4, where the strength of the composite is normalized by that of the non-reinforced alloy (σ_m), and plotted as a function of σ_m. The two lines in Fig. 4 show the calculated values of maximum composite strength (i.e. perfect bonding), which are in excellent agreement with the experimental data. Note that the aspect ratio of the whiskers equals the critical aspect ratio when $\sigma_m = 300$ MPa, and changes the slope of the calculated curves. Nevertheless the increases in strength are substantial for all values of matrix strength because of the very high strength of SiC whiskers.

4. Conclusions

1. Calculated values of the strength of metal matrix composites, reinforced with short fibres randomly oriented in three dimensions, are in good agreement with experimental data.

2. For a perfectly bonded composite, a 3D random system will produce a modest isotropic increase of strength.

3. If just the fibres perpendicular to the stress direction delaminate, the strength of a 3D composite is always less than the non-reinforced material.

4. The strength of a 3D random system is substantially less than that of the in-plane strength of a 2D random system.

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